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(3+1)-DIMENSIONAL EXPANDING UNIVERSE FROM A LORENTZIAN MATRIX MODEL FOR SUPERSTRING THEORY IN (9+1)-DIMENSIONS

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We study the Lorentzian version of the type IIB matrix model as a nonperturbative formulation of superstring theory in (9+1)-dimensions. Monte Carlo results show that not only space but also time emerges dynamically in this model. Furthermore, the real-time dynamics extracted from the matrices turns out to be remarkable: 3 out of 9 spatial directions start to expand at some critical time. This can be interpreted as the birth of our Universe.

Keywords: Superstring theory; matrix model; nonperturbative effects.

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1. Introduction

Fundamental questions concerning our Universe are:

- (i) Why does our Universe have (3+1)-dimensions?
- (ii) Why is it expanding ?

We provide explicit answers to these questions from a nonperturbative formulation of superstring theory in (9+1)-dimensions [1].

More specifically, we use the type IIB matrix model [2], which was proposed as a nonperturbative formulation of string theory based on type IIB superstrings in 10 dimensions. The Euclidean version of this model has been studied by various approaches. In particular, the Gaussian expansion method proposed in Ref. 3 was used to calculate the free energy of the $SO(d)$ symmetric vacua ($2 \leq d \leq 7$) and it was found that $d = 3$ gives the minimum [4]. This implies that the $SO(10)$ symmetry of the Euclidean model is spontaneously broken down to $SO(3)$. Moreover, the extent of space-time in the extended d directions and that in the shrunken $(10-d)$ directions turn out to have a finite ratio even in the large- N limit [4]. While these results reveal interesting dynamical properties of the Euclidean model, the connection to our real space-time is not very clear. This motivated us to consider the Lorentzian model [1].

Let us quote here an interesting statement made by Seiberg in his rapporteur talk entitled “Emergent Spacetime” [5] at the 23rd Solvay Conference in Physics

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in 2005: *Understanding how time emerges will undoubtedly shed new light on some of the most important questions in theoretical physics including the origin of the Universe.* Indeed, we will see that the emergent spacetime is naturally realized in the Lorentzian matrix model, and that the results can be interpreted as describing the birth of our Universe.

The rest of this article is organized as follows. In section 2 we review previous works on type IIB matrix model. In section 3 we define the Lorentzian matrix model. In section 4 we present our Monte Carlo results for the Lorentzian matrix model. Section 5 is devoted to a summary and discussions.

2. Previous works on type IIB matrix model

The action of the type IIB matrix model is given by [2]

$$S = S_b + S_f, \quad (1)$$

$$S_b = -\frac{1}{4} \text{tr} \left([A_\mu, A_\nu] [A^\mu, A^\nu] \right), \quad (2)$$

$$S_f = -\frac{1}{2} \text{tr} \left(\Psi_\alpha (\mathcal{C} \Gamma^\mu)_{\alpha\beta} [A_\mu, \Psi_\beta] \right), \quad (3)$$

where A_μ ($\mu = 0, \dots, 9$) and Ψ_α ($\alpha = 1, \dots, 16$) are $N \times N$ Hermitian matrices. The Lorentz indices μ and ν are contracted using the metric $\eta = \text{diag}(-1, 1, \dots, 1)$. The 16×16 matrices Γ^μ are ten-dimensional gamma matrices after the Weyl projection, and the unitary matrix \mathcal{C} is the charge conjugation matrix. The action has manifest $\text{SO}(9,1)$ symmetry, where A_μ and Ψ_α transform as a vector and a Majorana-Weyl spinor, respectively.

There are various evidences that the model gives a nonperturbative formulation of superstring theory [2].^a First of all, the action (1) can be viewed as a matrix regularization of the worldsheet action

$$S_{\text{Schild}} = - \int d^2\xi \sqrt{g} \left(\frac{1}{4} \{X_\mu, X_\nu\} \{X^\mu, X^\nu\} + \frac{1}{2} \Psi \mathcal{C} \Gamma^\mu \{X_\mu, \Psi\} \right) \quad (4)$$

of type IIB superstring theory in a particular gauge known as the Schild gauge. (The “Poisson brackets” are defined here by $\{f(\xi), g(\xi)\} \equiv \epsilon_{ij} \frac{\partial f}{\partial \xi_i} \frac{\partial g}{\partial \xi_j}$.) It has also been argued that configurations of block-diagonal matrices correspond to a collection of disconnected worldsheets with arbitrary genus. Therefore, instead of being equivalent just to the worldsheet theory, the large- N limit of the matrix model is expected to be a second-quantized theory of type IIB superstrings, which includes multi-string states. Secondly, D-branes are represented as classical solutions in the matrix model, and the interaction between them calculated at one loop reproduced correctly the known results from type IIB superstring theory [2]. Thirdly, one can derive the light-cone string field theory for the type IIB case from the matrix model [8]

^aThere are also other proposals for nonperturbative formulations of superstring/M theory based on matrix models such as Matrix Theory [6] and Matrix String Theory [7].

with a few assumptions. In the matrix model, one can define the Wilson loops, which can be naturally identified with the creation and annihilation operators of strings. Then, from the Schwinger-Dyson equations for the Wilson loops, one can actually obtain the string field Hamiltonian.

In all these connections to string theory, it is crucial that the model has two kinds of fermionic symmetries given by

$$\begin{cases} \delta^{(1)} A_\mu = i\epsilon_1 \mathcal{C} \Gamma_\mu \Psi , \\ \delta^{(1)} \Psi = \frac{i}{2} \Gamma^{\mu\nu} [A_\mu, A_\nu] \epsilon_1 , \end{cases} \quad (5)$$

$$\begin{cases} \delta^{(2)} A_\mu = 0 , \\ \delta^{(2)} \Psi = \epsilon_2 \mathbb{1} , \end{cases} \quad (6)$$

where $\mathbb{1}$ is the unit matrix. It also has the bosonic symmetry given by

$$\begin{cases} \delta^{(3)} A_\mu = c_\mu \mathbb{1} , \\ \delta^{(3)} \Psi = 0 . \end{cases} \quad (7)$$

Let us denote the generators of (5), (6) and (7) by $Q^{(1)}$, $Q^{(2)}$ and P_μ , respectively, and define their linear combinations

$$\tilde{Q}^{(1)} = Q^{(1)} + Q^{(2)} , \quad \tilde{Q}^{(2)} = i(Q^{(1)} - Q^{(2)}) . \quad (8)$$

Then, we find that the generators satisfy the algebra

$$[\epsilon_1 \mathcal{C} \tilde{Q}^{(i)}, \epsilon_2 \mathcal{C} \tilde{Q}^{(j)}] = -2\delta^{ij} \epsilon_1 \mathcal{C} \Gamma^\mu \epsilon_2 P_\mu , \quad (9)$$

where $i, j = 1, 2$. This is nothing but the ten-dimensional $\mathcal{N} = 2$ supersymmetry. It is known that field theories with this symmetry necessarily include gravity, which suggests that so does the type IIB matrix model. When we identify (9) with the ten-dimensional $\mathcal{N} = 2$ supersymmetry, the symmetry (7) is identified with the translational symmetry in ten dimensions, which implies that the eigenvalues of A_μ should be identified with the coordinates of ten-dimensional space-time. This identification is consistent with the one adopted in stating the evidences listed in the previous paragraph, and shall be used throughout this article as well.

An interesting feature of the type IIB matrix model is that the space-time itself is treated as a part of dynamical degrees of freedom in the matrices. Therefore, it is possible that a four-dimensional space-time is generated dynamically in this model. This issue has been studied in the Euclidean version of the model [3, 4, 9–23], which can be obtained from (1) by making a Wick rotation

$$A_0 \mapsto iA_{10} , \quad \Gamma^0 \mapsto -i\Gamma^{10} . \quad (10)$$

Note that the Euclidean model has $\text{SO}(10)$ symmetry instead of $\text{SO}(9,1)$.

In order to discuss the spontaneous symmetry breaking (SSB) of $\text{SO}(10)$ in the large- N limit, we consider the “moment of inertia” tensor [9, 10]

$$T_{\mu\nu} = \frac{1}{N} \text{tr} (A_\mu A_\nu) , \quad (11)$$

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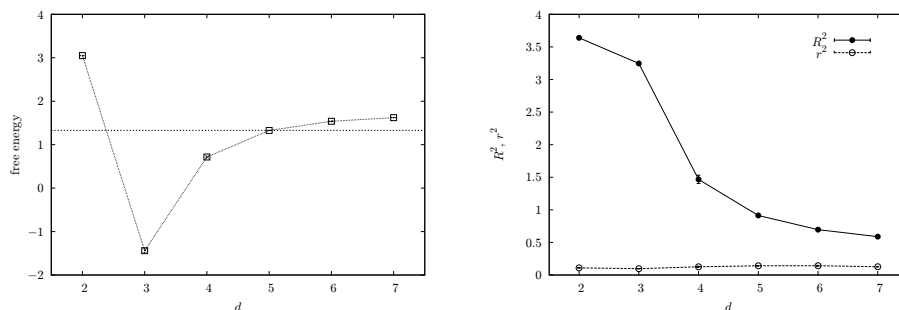


Fig. 1. (Left) The free energy of the SO(d) symmetric vacuum is plotted against d . The horizontal line represents the value $f = \log 8 - \frac{3}{4} = 1.32944\dots$ obtained from the conjecture by Krauth, Nicolai and Staudacher [24]. (See footnote b.) The dotted line connecting the data points is drawn to guide the eye. (Right) The extent of space-time R^2 and r^2 in the extended and shrunken directions, respectively, are plotted against d . The solid and dashed lines connecting the data points are drawn to guide the eye.

which is a 10×10 real symmetric tensor. We denote its eigenvalues as λ_j ($j = 1, \dots, 10$) with the specific order

$$\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_{10} . \quad (12)$$

If the SO(10) symmetry is not spontaneously broken, the expectation values $\langle \lambda_j \rangle$ ($j = 1, \dots, 10$) should be all equal in the large- N limit. The dynamical generation of d -dimensional space-time corresponds to an SO(d) symmetric vacuum, in which the expectation values $\langle \lambda_j \rangle$ ($j = 1, \dots, d$) are equal in the large- N limit, but the remaining ones $\langle \lambda_j \rangle$ ($j = d + 1, \dots, 10$) are much smaller.

Let us show recent results obtained by the Gaussian expansion method [4]. In Fig. 1 (Left) we plot the free energy of the SO(d) symmetric vacuum for $2 \leq d \leq 7$ at order 3 of the expansion. The result decreases monotonically as d decreases from 7 to 3, and it becomes much larger for $d = 2$. Thus, the SO(3) symmetric vacuum gives the smallest free energy, which suggests the SSB of SO(10) down to SO(3).^b

Let us discuss the results for the extent of space-time represented by the eigenvalues (12). In Fig. 1 (Right) we plot the result for the extended directions (R^2) and the shrunken directions (r^2) for each d . We find that r^2 stays almost constant at $r^2 = 0.1 \sim 0.15$, which seems to be universal for all the SO(d) symmetric vacua with $2 \leq d \leq 7$. On the other hand, the results for R^2 are found to be larger for smaller d . Thus the extent of space-time seems to be finite in all directions. While the observed SSB of SO(10) is an interesting dynamical property of the Euclidean

^bThe d -dependence of the free energy is quite analogous to the one observed in the six-dimensional model [25]. There the value of the free energy tends to decrease slightly as one goes from order 3 to order 5. Considering such artifacts due to truncation, we speculate that the Krauth-Nicolai-Staudacher conjecture [24] actually refers to the partition function for the SO(10) symmetric vacuum. See Fig. 1 (Left).

model, the connection to the real space-time is unclear.

3. Defining the Lorentzian matrix model

The most crucial difference between the Euclidean and Lorentzian models is the bosonic part of the action (2). In the Euclidean model, it becomes positive semi-definite

$$S_b \propto \text{tr} (F_{\mu\nu})^2, \quad (13)$$

where we have defined Hermitian matrices $F_{\mu\nu} = i[A_\mu, A_\nu]$. There is a flat direction corresponding to $[A_\mu, A_\nu] \sim 0$, but it is lifted up by quantum effects from fermionic zero modes [9]. Thus the model is well defined without any cutoff [24, 26].

In the Lorentzian model, the bosonic action can be decomposed into two terms

$$S_b \propto \text{tr} (F_{\mu\nu} F^{\mu\nu}) = -2 \text{tr} (F_{0i})^2 + \text{tr} (F_{ij})^2 \quad (14)$$

with opposite signs. The model looks extremely unstable, and hence no one has ever dared to study this model seriously.

Another important difference between the Euclidean and Lorentzian models is the Pfaffian $\text{Pf}\mathcal{M}(A)$ obtained by integrating out fermions. In the Euclidean model, the Pfaffian is complex, and the phase plays a crucial role in SSB of $\text{SO}(10)$ [13–15]. But it also makes Monte Carlo studies extremely difficult [16–18] due to the so-called sign problem. In the Lorentzian model, the Pfaffian is real. This is a good news for Monte Carlo studies, but we also lose a source of SSB.

In the case of Euclidean model, the partition function was defined by

$$Z_E = \int dA d\Psi e^{-S} = \int dA e^{-S_b} \text{Pf}\mathcal{M}(A). \quad (15)$$

In the case of Lorentzian model, we define the partition function of the Lorentzian model by [1]

$$Z_L = \int dA d\Psi e^{iS} = \int dA e^{iS_b} \text{Pf}\mathcal{M}(A). \quad (16)$$

Here we have replaced the “Boltzmann weight” e^{-S} used in the Euclidean model by e^{iS} . This is theoretically motivated from the connection to the worldsheet theory (4). When we make an inverse Wick rotation, we need to change the worldsheet coordinate $\xi_0 \equiv -i\xi_2$ as well as the target-space coordinates. Applying the mapping rule from fields to matrices [2], we obtain (16).

Note that e^{iS_b} in the partition function (16) is a phase factor just as in the path-integral formulation of quantum field theories in Minkowski space. This may give rise to the sign problem when one tries to study the model by Monte Carlo simulation. In the present case, however, the sign problem can actually be circumvented in the following way. The crucial point here is that the action of the type IIB matrix model is homogeneous with respect to the matrices. Under the scale transformation

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$A_\mu \mapsto \rho A_\mu$, each part of the partition function (16) transforms as

$$S_b \mapsto \rho^4 S_b , \quad (17)$$

$$dA \mapsto \rho^{10(N^2-1)} dA , \quad (18)$$

$$\text{Pf}\mathcal{M}(A) \mapsto \rho^{8(N^2-1)} \text{Pf}\mathcal{M}(A) . \quad (19)$$

Integrating out the scale factor of the bosonic matrices first,^c one essentially converts the phase factor e^{iS_b} into a constraint $S_b \approx 0$. (Such a constraint is analogous to the one that appeared in the model inspired by space-time uncertainty principle [27].)

It turns out that the integration over A_μ in (16) is divergent, and we need to introduce two constraints

$$\frac{1}{N} \text{tr} (A_0)^2 \leq \kappa \frac{1}{N} \text{tr} (A_i)^2 , \quad (20)$$

$$\frac{1}{N} \text{tr} (A_i)^2 \leq L^2 . \quad (21)$$

This is in striking contrast to the Euclidean model, in which the partition function is shown to be finite without any regularization [24, 26].

Without loss of generality, we set $L = 1$ in (21), and thus we arrive at the model

$$Z = \int dA \delta \left(\frac{1}{N} \text{tr} (F_{\mu\nu} F^{\mu\nu}) \right) \text{Pf}\mathcal{M}(A) \delta \left(\frac{1}{N} \text{tr} (A_i)^2 - 1 \right) \theta \left(\kappa - \frac{1}{N} \text{tr} (A_0)^2 \right) , \quad (22)$$

where $\theta(x)$ is the Heaviside step function. Since the Pfaffian $\text{Pf}\mathcal{M}(A)$ is real in the present Lorentzian case, the model (22) can be studied by Monte Carlo simulation without the sign problem.^d Note that this is usually not the case for quantum field theories in Minkowski space.

4. Monte Carlo results for the Lorentzian matrix model

We perform Monte Carlo simulation^e of the model (22) by using the RHMC algorithm [29].

It turned out that not only space but also time emerges dynamically in the Lorentzian matrix model. We found that the eigenvalue distribution of A_0 extends in the large- N limit. Here, supersymmetry of the model plays a crucial role. If we omit fermions, the eigenvalue distribution has a finite extent, and the cutoff (20) in the temporal direction is actually not needed [30].

In order to extract the “time evolution”, we diagonalize A_0 , and define the eigenvectors $|t_a\rangle$ corresponding to the eigenvalues t_a of A_0 ($a = 1, \dots, N$) with the specific order $t_1 < \dots < t_N$. The spatial matrix in this basis $\langle t_a | A_i | t_b \rangle$ is not

^cThe same procedure was also used in Ref. 24 for simulating the Euclidean model.

^dStrictly speaking, the Pfaffian can flip its sign, but we find that the configurations with positive Pfaffian dominates as N is increased. Hence, we just take the absolute value of the Pfaffian in actual simulation.

^eFor a recent review on various applications of Monte Carlo simulation to string theory, see Ref. 28.

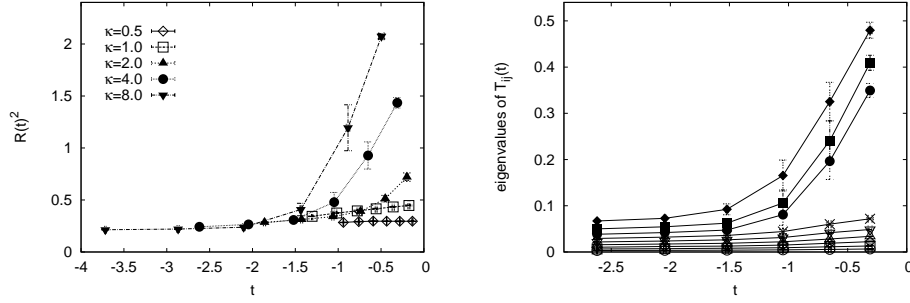


Fig. 2. (Left) The extent of space $R(t)^2$ with $N = 16$ and $n = 4$ is plotted as a function of t for five values of κ . The peak at $t = 0$ starts to grow at some critical κ . (Right) The 9 eigenvalues of $T_{ij}(t)$ with $N = 16$ and $n = 4$ are plotted as a function of t for $\kappa = 4.0$. After the critical time t_c , 3 eigenvalues become larger, suggesting that the $SO(9)$ symmetry is spontaneously broken down to $SO(3)$.

diagonal, but it turns out that the off-diagonal elements decrease rapidly as one goes away from a diagonal element. This motivates us to define $n \times n$ matrices

$$\bar{A}_i^{(ab)}(t) \equiv \langle t_{\nu+a} | A_i | t_{\nu+b} \rangle \quad (23)$$

with $1 \leq a, b \leq n$ and $t = \frac{1}{n} \sum_{a=1}^n t_{\nu+a}$ for $\nu = 0, \dots, (N-n)$. These matrices represent the 9d space structure at fixed time t . (This point of view can be justified in the large- N limit, in which more and more eigenvalues of A_0 appear around some value t within a fixed interval δt .) The block size n should be large enough to include non-negligible off-diagonal elements. In Fig. 2 (Left) we plot the extent of space

$$R(t)^2 \equiv \frac{1}{n} \text{tr} \bar{A}_i(t)^2 \quad (24)$$

for $N = 16$ and $n = 4$. Since the result is symmetric under the time reflection $t \rightarrow -t$ as a consequence of the symmetry $A_0 \rightarrow -A_0$, we only show the results for $t < 0$. There is a critical κ , beyond which the peak at $t = 0$ starts to grow.

Next we study the SSB of the $SO(9)$ symmetry. As an order parameter, we define the 9×9 (positive semi-definite) real symmetric tensor

$$T_{ij}(t) = \frac{1}{n} \text{tr} \left\{ \bar{A}_i(t) \bar{A}_j(t) \right\}. \quad (25)$$

The 9 eigenvalues of $T_{ij}(t)$ are plotted against t in Fig. 2 (Right) for $\kappa = 4.0$. We find that 3 largest eigenvalues of $T_{ij}(t)$ start to grow at the critical time t_c , which suggests that the $SO(9)$ symmetry is spontaneously broken down to $SO(3)$ after t_c . Note that $R(t)^2$ is given by the sum of 9 eigenvalues of $T_{ij}(t)$.

It turned out that one can remove the infrared cutoffs κ and L in the large- N limit in such a way that $R(t)$ scales. This can be done in two steps. (i) First we send κ to ∞ with N as $\kappa = \beta N^p$ ($p \simeq \frac{1}{4}$) [30]. The scaling behavior is clearly seen in Fig. 3 (Left). The scaling curve of $R(t)$ one obtains in this way depends on β . (ii) Next we send β to ∞ with L . The two limits correspond to the continuum limit

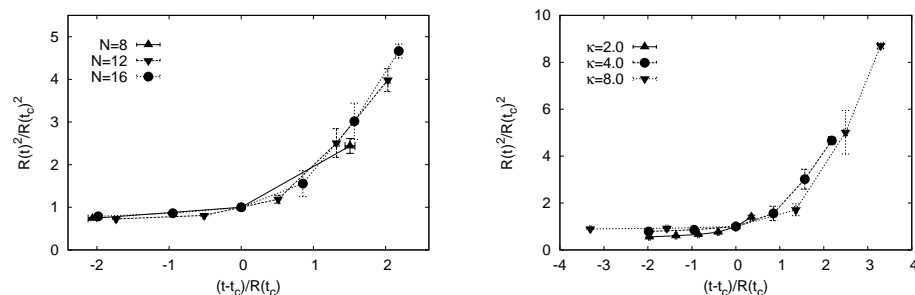


Fig. 3. (Left) The extent of space $R(t)^2$ for $\kappa = \beta N^{1/4}$ is plotted for $N = 8, 12, 16$ with $\beta = 2$. We plot the results against the shifted time $t - t_c$ in units of the size of the universe $R(t_c)$ at the critical time. (Right) Similar plot for fixed $N = 16$ with $\kappa = 2, 4, 8$.

and the infinite volume limit, respectively, in quantum field theory. Thus the two constraints (20), (21) can be removed in the large- N limit, and the resulting theory has no parameter other than one scale parameter.

Let us discuss the second limit (ii) in more detail. We find that the inequality (21) is actually saturated for the dominant configurations. Therefore, one only has to make the rescaling $A_\mu \mapsto LA_\mu$ in order to translate the configurations in the model (22) as those in the original partition function. It turns out that $R(t)$ for the rescaled configurations scales in β by tuning L and shifting t appropriately. In order to see this, it is convenient to choose L so that $R(t)$ at the critical time $t = t_c$ becomes unity, and to shift t so that the critical time comes to the origin. Then $R(t)$ with increasing β extends in t in such a way that the results at smaller $|t|$ scale. This is demonstrated in Fig. 3 (Right), where we find a reasonable scaling behavior for $N = 16$ with $\kappa = 2.0, 4.0, 8.0$. Note, in particular, that the extent of “time” increases as κ is increased, which is not the case in the bosonic model [30]. As we mentioned earlier, supersymmetry plays a crucial role for the “emergent time” in the Lorentzian matrix model.

The SSB of $SO(9)$ looks mysterious at first sight, but we can actually understand the mechanism quite intuitively. Let us consider the case in which κ is large. Then the first term of (14) becomes a large negative value, and therefore the second term has to become large in order to make (14) zero as required in (22). Due to the constraint $\frac{1}{N} \text{tr}(A_i)^2 = 1$, however, it is more efficient to maximize the second term of (14) at some fixed time. The system actually chooses the middle point $t = 0$, where the suppression on A_i from the first term of (14) becomes the least. This explains why the peak of $R(t)$ at $t = 0$ grows as we increase κ .

Let us then consider a simplified question: what is the configuration of A_i which gives the maximum $\frac{1}{N} \text{tr}(F_{ij})^2$ with fixed $\frac{1}{N} \text{tr}(A_i)^2 = 1$. Using the Lagrange multiplier λ , we maximize the function $G = \text{tr}(F_{ij})^2 - \lambda \text{tr}(A_i)^2$. Taking the derivative with respect to A_i , we obtain $2[A_j, [A_j, A_i]] - \lambda A_i = 0$. This equation can be solved

if $A_i = \chi L_i$ for $i \leq d$, and $A_i = 0$ for $d < i \leq 9$, where L_i are the representation matrices of a compact semi-simple Lie algebra with d generators. Clearly d should be less than or equal to 9. It turns out that the maximum of $\frac{1}{N} \text{tr}(F_{ij})^2$ is achieved for the $\text{SU}(2)$ algebra, which has $d = 3$, with L_i being the direct sum of the spin- $\frac{1}{2}$ representation and $(N - 2)$ copies of the trivial representation. This implies the SSB of $\text{SO}(9)$ down to $\text{SO}(3)$. The SSB can thus be understood as a classical effect in the $\kappa \rightarrow \infty$ limit. When we tune κ with increasing N as described above, quantum effects become important. We have confirmed [30] that the $n \times n$ matrix $Q = \sum_{i=1}^9 \bar{A}_i(t)^2$ has quite a continuous eigenvalue distribution, which implies that the space is not like a two-dimensional sphere as one might suspect from the classical picture.

5. Summary

We have studied the type IIB matrix model as a nonperturbative formulation of superstring theory in (9+1)-dimensions. Unlike previous works, we made the Lorentzian matrix model well-defined by introducing infrared cutoffs instead of making a Wick rotation.

Monte Carlo studies of the Lorentzian matrix model revealed the following non-trivial facts:

- The infrared cutoffs can be removed in the large- N limit.
- The theory thus obtained has no dimensionless parameter, supporting the validity of the model as a nonperturbative formulation of superstring theory.
- “Time” emerges dynamically thanks to supersymmetry.
- “Time evolution” of 9d space emerges.
- 3 out of 9 spatial directions start to expand at some “critical time”, after which the $\text{SO}(9)$ symmetry of the space is broken spontaneously down to $\text{SO}(3)$.

All these results for the Lorentzian matrix model suggest that (3+1)-dimensional expanding universe emerges dynamically from superstring theory if the theory is treated nonperturbatively. This may be contrasted with the quantum cosmology in the early 80s [32–34] that aimed at describing the birth of the universe within the mini-superspace approximation.^f Note also that the picture suggested here is quite different from that in (perturbative) superstring theory, where space-time with various dimensions can be obtained by compactification or by using D-brane backgrounds.

The rapid expansion of the three-dimensional space observed in Monte Carlo simulation may be interpreted as the beginning of inflation. It would be interesting to investigate the microscopic origin of the inflation along this line.

^fMore recently, a nonperturbative approach to quantum gravity has been pursued using the causal dynamical triangulation [31]. For earlier works that put forward the idea to use matrices for cosmology, see Refs. 35, 36. See also Refs. 37, 38 for related works on emergent gravity.

The mechanism of the SSB relies crucially on noncommutativity of the space-time represented by 10 bosonic matrices. Therefore, an important issue is whether the usual commutative space-time appears at later times. We addressed this issue by studying the classical equations of motion, which are expected to be valid at late times [39, 40]. There are actually infinitely many solutions representing commutative (3+1)-dimensional space-time. Moreover, we found a simple solution with an expanding behavior, which naturally solves the cosmological constant problem [40]. We consider that there exists a unique solution of this kind that dominates the partition function of the matrix model at late times. By pursuing this direction further, it would be possible to understand the origin of dark energy found in the present cosmological observations and to predict the fate of our Universe.

Since superstring theory is not only a theory of quantum gravity but also a theory of all the matters and the fundamental interactions among them, it would be interesting to see how the Standard Model appears at late times in the Lorentzian matrix model. Finding solutions to the classical equation of motions [41–45] and performing perturbative expansion around them would be an important direction as an approach complementary to Monte Carlo simulation.

Recently, we have proposed an explicit procedure to identify the local fields corresponding to the massless modes that appear at late times [46]. The basic assumption is that the low-lying spectrum is essentially determined by the Nambu-Goldstone modes (and their extension) associated with the SSB of the (9+1)-dimensional Poincare symmetry and supersymmetry. The local field theory obtained in this way below the Planck scale has interesting generic features. It is a grand unified theory with the gauge group $SU(k)$, and all the matter fields are in the adjoint representation. The grand unified theory with $k = 8$, for instance, can accommodate all the Standard Model particles. As the space-time dimensionality seems to be uniquely determined by nonperturbative dynamics of superstring theory, we consider it quite conceivable that the Standard Model emerges uniquely from this top-down approach. Further investigations are clearly worth while.

To conclude, we believe that the Lorentzian matrix model, as a correct and tractable nonperturbative formulation of superstring theory, provides totally new perspectives in both particle physics and cosmology.

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